Letter

The geometric phase: interferometric observations with white light

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Abstract. Observations on white-light interference fringes show that the effects due to the introduction of a variable geometric phase, which is independent of the wavelength, differ significantly from those due to a change in the optical path difference.

1. Introduction

Berry [1] has shown that the wavefunction of a quantum system may acquire an additional phase factor (the geometric phase) when the system is taken around a circuit in parameter space. In the case of photons, such a phase shift can be produced by a cyclic change in their state of polarization (the Pancharatnam phase) [2–5]. Several experimental measurements of the Pancharatnam phase have been made [6–11] with monochromatic light; some preliminary observations have also been made using a simple white-light interferometer [12]. This letter presents quantitative measurements on the interference fringes obtained with white light in an interferometer using an achromatic phase shifter operating on the Pancharatnam phase.

2. Optical system

The apparatus used in these experiments is shown schematically in figure 1. Light from a tungsten lamp, linearly polarized at $45°$ to the plane of the figure by a polarizer P_1, is divided at a polarizing beam splitter into two orthogonally polarized beams that traverse the same closed triangular path in a Sagnac interferometer in opposite directions. A second polarizer P_2, with its axis at $45°$ to the plane of the figure, brings the two beams leaving the interferometer into a condition to interface.

The two beams in this interferometer always emerge parallel to one another and can be made to coincide by adjusting the beam splitter and the mirrors. This adjustment can be made conveniently with a laser as the light source. Interference fringes can then be obtained quite easily with the white-light source.

Since both the beams traverse the same optical circuit, the optical path difference is always equal to zero at the centre of the field of view. When a small lateral shear was introduced between the beams, an interference pattern consisting of equally spaced, straight fringes was formed on the charge-coupled device (CCD) array placed in the focal plane of the imaging lens L_2. This interference pattern could be viewed conveniently on the television monitor; the intensity distribution in the pattern could also be recorded and stored for further analysis on the computer.
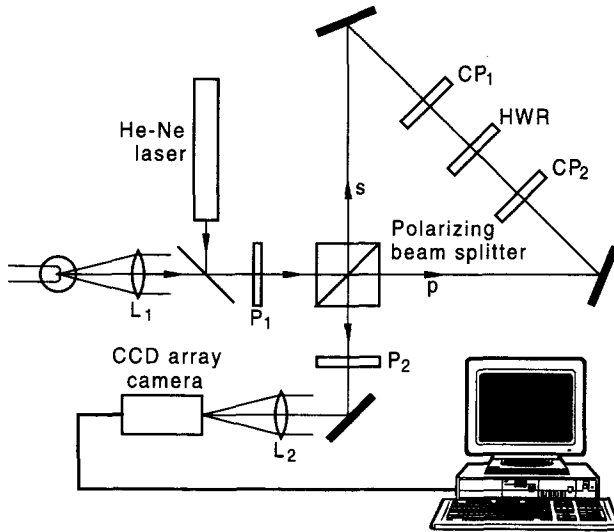
Figure 1. Schematic diagram of the optical system of the interferometer.

The phase difference between the beams was varied by a system that operated on the geometric phase, consisting of a half-wave retarder (HWR), which could be rotated by known amounts, located between two circular polarizers CP_1 and CP_2. Since the retardation produced by a single birefringent plate is inversely proportional to the wavelength, we used for CP_1 and CP_2 a combination of a half-wave plate and a quarter-wave plate which yields an achromatic circular polarizer [13], and for the HWR a sandwich consisting of two quarter-wave plates and a half-wave plate, which is a good approximation to an achromatic HWR [14].

3. Theory

The phase change introduced by moving a polarization state around a circuit on the Poincaré sphere [15, 16] is equal to half the solid angle subtended by the circuit at the centre of the sphere. In the present arrangement the linearly (s-)polarized beam reflected by the polarizing beam splitter passes through the circular polarizer CP_1, the HWR and the circular polarizer CP_2, in that order and, as shown in figure 2, the polarization state of this beam traces out the closed circuit $A_1SA_2NA_1$ on the Poincaré sphere. If the HWR is set with its principal plane at an angle $+\theta$ to the principal planes of CP_1 and CP_2, the transmitted beam acquires a phase shift of 2θ. The transmitted (p-polarized) beam traverses the interferometer in the opposite direction, and its polarization state traces out the path $B_1SB_2NB_1$ on the Poincaré sphere. Since the circuit $B_1SB_2NB_1$ is identical with the circuit $A_1SA_2NA_1$ but is traversed in the opposite sense, this beam experiences a phase shift of -2θ. These operations on the geometrical phase lead to an additional phase difference $\Delta\varphi_G = 4\theta$ between the two beams in the interferometer, without introducing any change in the optical paths. Because of the topological nature of the geometric phase, the phase difference that can be introduced in this manner is unbounded and its sign can be reversed [10].
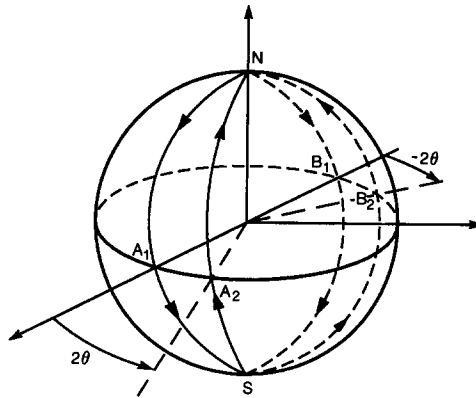
Figure 2. Poincaré sphere representation of the closed paths traversed by the polarization states of the two beams in the interferometer.

4. Experimental results

With monochromatic illumination, rotation of the HWR resulted in a continuous movement of the interference fringes across the field of view. When the direction of rotation of the HWR was reversed, the fringes moved continuously in the opposite direction.

With white light, the interference pattern seen without the geometrical phase shifter in the optical path consisted of a central black fringe flanked on each side by a white fringe and a few coloured fringes whose contrast decreased rapidly with their distance from the central fringe. With the geometric phase shifter in the optical path, a rotation of the HWR produced in this case also a movement of the fringes across the field of view, in the direction corresponding to the sense of rotation. A rotation of the HWR by $\pm 45°$ caused the central black fringe to be replaced by a white fringe, flanked on each side by a black fringe, while a rotation of the HWR by $\pm 90°$ brought back the central black fringe. However, as can be seen from the profiles presented in figure 3, showing the intensity along a line at right angles to the fringes, the position of the fringe envelope in the field of view did not change.

5. Discussion

This interferometer produces interference fringes of equal inclination, localized at infinity [17]. If the lateral shear between the beams in the plane of the figure is s, the optical path difference between the beams at a distance x from the centre of the field of view in this plane is

$$p = \frac{s}{f} x, \qquad (1)$$

where f is the focal length of the imaging lens L_2. In the absence of the geometric phase shifter, successive intensity minima are defined by the condition
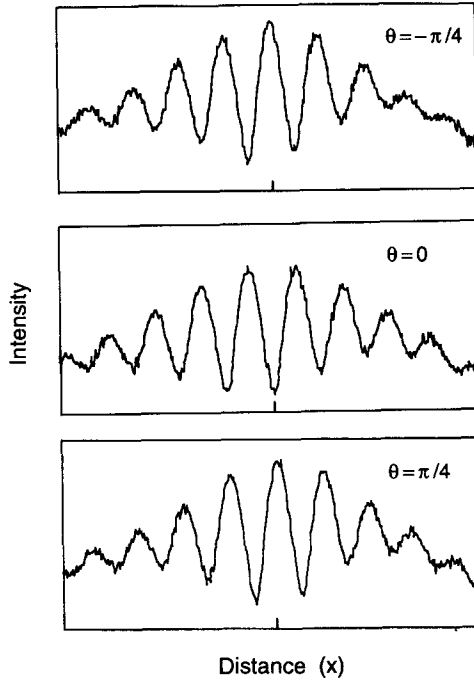
$$\Delta\varphi_D = \frac{2\pi}{\lambda} p = 2m\pi, \qquad (2)$$

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Figure 3.   Intensity profiles across the interference patterns recorded with white light for different angular settings θ of the HWR.

where m is an integer. Successive minima corresponding to any wavelength are separated by a distance

$$\Delta x = \frac{\lambda f}{s}, \tag{3}$$

which is directly proportional to the wavelength.

With the geometric phase shifter in position, the total phase difference between the beams at a point at a distance x from the centre of the field of view is

$$\Delta\varphi_{\text{total}} = \Delta\varphi_{\text{D}} + \Delta\varphi_{\text{G}}. \tag{4}$$

Accordingly, the effect of varying the geometric phase is to move the interference fringes across the field of view. However, unlike the dynamic phase $\Delta\varphi_{\text{D}}$ due to a change in the optical path, which depends on the wavelength, the geometric phase $\Delta\varphi_{\text{G}}$ is independent of the wavelength. The overall effect of a change in the geometric phase of $2m\pi$ is therefore to move the fringes formed at any wavelength by exactly one fringe spacing, so that the interference pattern produced with white light returns to its original configuration. In particular, at the centre of the fringe envelope, which corresponds to the condition that the dynamic phase $\Delta\varphi_{\text{D}} = 0$, a variation in the geometric phase $\Delta\varphi_{\text{G}}$ has the same effect on the interference patterns produced by all the wavelengths. As a result, while the intensity at this point varies between its minimum and maximum values, the position of the centre of the fringe envelope in the field of view remains unchanged.

6.   Conclusions

These observations confirm that the effects due to the introduction of a variable geometric phase, which is independent of the wavelength, are not the same as those

due to a change in the optical path difference, which produces a variation in the dynamic phase that is inversely proportional to the wavelength. They also suggest that the geometric phase, even though it is unbounded, can only be measured to modulo 2π.

References

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