Effect of numerical aperture on interference fringe spacing

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The effect of numerical aperture on the fringe spacing in interferometry is analyzed by the use of wave optics. The results are compared with published experimental results, and the influence of apodization of the wave front is discussed. The effects of central obscuration and surface tilt are also considered.

1. Introduction

In interferometric measurements of surface profiles, the numerical aperture of the system has been shown to affect the spacing of the fringes, and hence the calibration of the method. Various theories for this effect have been described.[1–5] Most of the theories are simplistic and based on geometric optics, although that of Schulz,[5] based on averaging over fringes, agrees with the present results. A theory for the general axial phase variation has also been previously presented.[6] This last publication was primarily concerned with confocal imaging, but the results are equally applicable to conventional interferometric systems. Various presentations of experimental investigations have also been given.[7–11]

An interferometric image consists of three terms. In addition to the interferometric term, there is a conventional intensity image and a constant reference beam term. The interference term can be extracted by the use of phase shifting or heterodyning. In general, the conventional intensity term is partially spatially coherent, whereas the intensity term for a confocal system is fully spatially coherent.[12] In confocal microscopes one can extract the interference term by changing the modulus of the reference beam rather than the phase.[13]

It is found that the phase of the interference term varies with surface height in a fashion that depends on the numerical aperture. Far from focus, the average gradient of the phase variation with height tends to a value that is independent of numerical aperture, but near the focal plane the phase gradient is reduced. This effect is closely connected with the phase anomaly in the focal region.

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2. Effects of Aperture

We consider an interferometric system with monochromatic illumination. In a Linnik microscope system, in which the reference mirror and matched objective are moved to adjust the phase of the reference beam, the measured phase determined by phase stepping is simply the phase of the signal beam. This is also the case in a heterodyne system, in which a phase modulator is placed in the reference beam path, or a confocal system, in which a matched objective is not necessary. For a level, plane specimen, the amplitude of the interferometric term can be determined simply by integration, with appropriate weighting, over the amplitude of the angular spectrum of plane-wave components reflected by the surface. We have[6]

$$U(z) = \int_0^{\alpha} P_1(\theta)P_2(\theta)R(\theta)\exp(-2ikz\cos\theta)\sin\theta \mathrm{d}\theta, \quad (1)$$

where θ is the angle of a ray relative to the optic axis, $P_{1,2}(\theta)$ are the pupil functions of the illuminating and collecting lenses, including a $\cos^{1/2}\theta$ weighting for an aplanatic system (i.e., an aberration-free system satisfying Abbe's sine condition), $R(\theta)$ is the reflectivity of the surface, z is its displacement from the focal plane, $k = 2\pi/\lambda$, and $\sin\alpha$ is the numerical aperture of the system.

The phase gradient predicted by Eq. (1) depends on the apodization of the system. To investigate the effects of apodization we must make a choice of an appropriate form for the apodization function. Possibilities included Legendre polynomials or various trigonometric functions. We choose to assume an

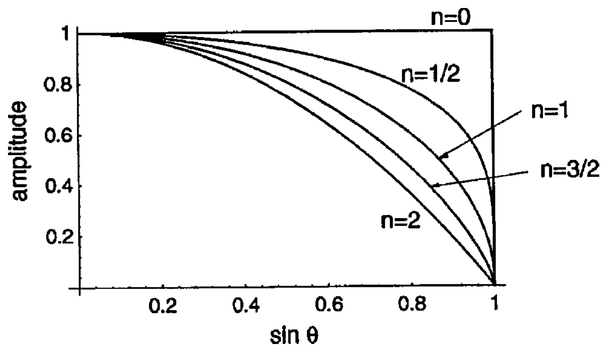

Fig. 1. Variation in the amplitude of the apodization for different values of parameter n. The value $n = \frac{1}{2}$ applies for a perfect system satisfying the sine condition.

apodization

$$P(\theta) = \cos^n \theta, \tag{2}$$

so that $n = \frac{1}{2}$ corresponds to the perfect aplanatic case. The value $n = 0$ corresponds to the case of a constant angular variation (the Herschel condition), and higher values of n correspond to apodization that falls off faster with angle. The resultant apodization functions are illustrated in Fig. 1. It should be noted that real microscope objectives exhibit an apodization effect[13–15] that results from Fresnel reflections at the surfaces of the optical elements.

For a perfect reflector and an aberration-free aplanatic system we thus have

$$U(z) = \int_0^\alpha \exp(-2ikz \cos \theta) \sin \theta \cos \theta d\theta. \tag{3}$$

In the region close to focus we can expand the exponential function as a power series, keeping just the first two terms. These can then be integrated directly to give the phase gradient in this region. After integration, the ratio of $2kz$ to the phase, called the NA factor, is given by

$$f = \left(\frac{2n+2}{2n+1}\right)\left(\frac{1-\cos^{2n+1}\alpha}{1-\cos^{2n+2}\alpha}\right). \tag{4}$$

This expression agrees with that of Schulz[5] for $n = 0$ and $n = \frac{1}{2}$. The factor is plotted in Figs. 2 and 3.

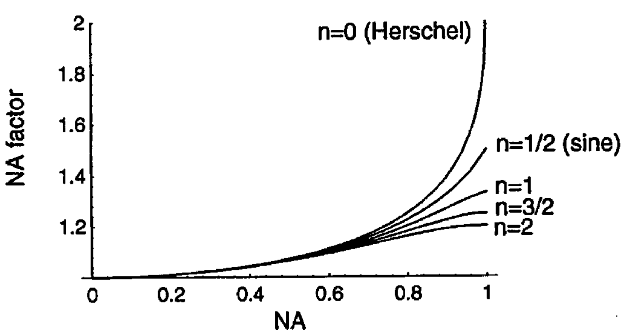

Fig. 2. Variation in the NA factor (the height change per half-fringe) with numerical aperture for different values of parameter n.

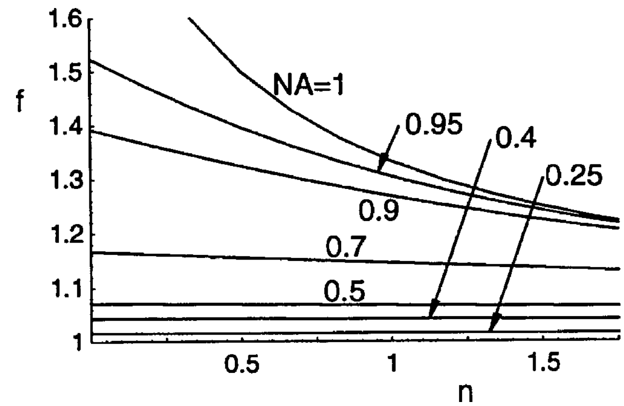

Fig. 3. Variation in NA factor f with parameter n for different numerical apertures.

It is greater than unity, indicating that the fringes are more widely spaced than those for a low-aperture system. In Table 1 values for particular numerical apertures are given and compared with the experimental results of Biegen[10] and Creath.[11] The results of Biegen agree well with the theoretical results, but for higher numerical apertures the values predicted for a perfect aplanatic system are larger than those observed in practice. For example, for a numerical aperture of 0.95, we need to assume a value of $n = 0.6$ to predict the observed behavior.

The phase variation with axial position has been presented in an analytical form for an aplanatic system.[6] The phase can be differentiated to calculate factor f for an arbitrary distance from focus. For $n = 0$, factor f is independent of focus position, but for other values of n it decreases away from focus (Fig. 4). It can become less than unity for axial positions that correspond to regions for which the fringe visibility is small. Thus for measurements of large phase steps, an effective factor should be employed. In particular, for high apertures the region over which the factor is constant is small. For example, for a numerical aperture of 0.95 in an aplanatic system, the factor drops by 1% for a distance from the focal plane of only approximately $\lambda/4$. In principle, one can avoid this complication by bringing the surface to the focal plane. Schulz[2] considered the measurement phase errors resulting from the naive application of phase-measuring interferometry and recommended a calibration procedure

Table 1. Values for Particular Numerical Apertures

	Theoretical			Observed	
NA	$n = 0$ (Herschel)	$n = \frac{1}{2}$ (sine)	$n = 1$	Ref. 11	Ref. 10
0.10	1.003	1.003	1.003	1.003	1.003
0.25	1.016	1.016	1.016	1.007	1.016
0.40	1.044	1.043	1.042	1.024	1.021
0.50	1.072	1.070	1.068	1.036	1.057
0.90	1.393	1.325	1.269	1.215	1.258
0.95	1.524	1.396	1.305	1.228	1.337

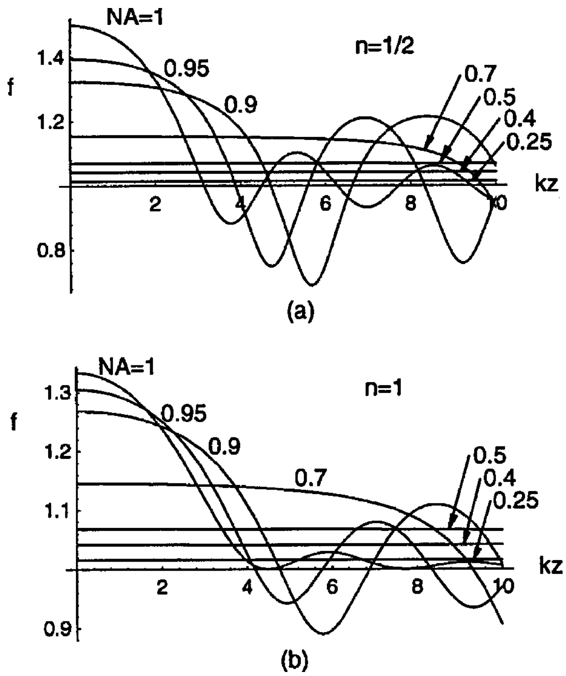

Fig. 4. Variation in NA factor f with distance from the focal plane: (a) $n = \frac{1}{2}$, (b) $n = 1$. For $n = 0$, factor f is independent of focus position.

that used a large number of closely spaced height gauges.

3. Effect of Central Obscuration

The results can readily be extended to the case of an objective with a central obscuration, such as a Mirau objective. Introducing the obscuration ratio

$$\varepsilon = \frac{\sin \alpha_0}{\sin \alpha}, \tag{5}$$

where $\sin \alpha_0$ is the numerical aperture of the central obscuration, we can simply write

$$f = \left(\frac{2n + 2}{2n + 1}\right)\left(\frac{\cos^{2n+1} \alpha_0 - \cos^{2n+1} \alpha}{\cos^{2n+2} \alpha_0 - \cos^{2n+2} \alpha}\right). \tag{6}$$

For a perfect aplanatic system the behavior is illustrated in Fig. 5. The obscuration has the effect of further increasing the fringe spacing; the results are similar to those presented by Schulz.[5]

4. Effect of Surface Tilt

The phase gradient can be predicted by a simple model. The amplitude image of a surface can be evaluated as the Fourier transform of the product of the three-dimensional coherent transfer function of the system and the three-dimensional object spectrum. The application of the well-known moment-derivative property of Fourier transforms gives the phase gradient (along the optical axis), which is simply proportional to the distance from the origin of the center of gravity of this product projected onto the axial spatial frequency axis. This simple relationship is true only for real transfer functions, so it cannot be applied to the investigation of the effects of aberrations.

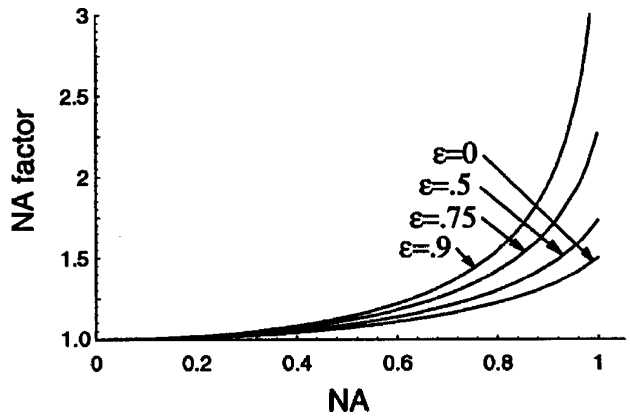

Fig. 5. Effect of a central obscuration on the NA factor for a perfect system satisfying sine condition $n = \frac{1}{2}$.

The object spectrum for a perfectly reflecting surface (i.e., the reflection coefficient is unity at all angles) is zero, except along a line in spatial frequency space normal to the surface. Along this line, the value of the spectrum increases linearly with the spatial frequency.[16–17] The surface thus behaves differently from a single plane, which does not reflect equally at all angles. The coherent transfer function for reflection imaging has been presented elsewhere for a scalar, high-angle theory.[18] For a system satisfying the sine condition, one can express the coherent transfer function analytically in terms of elliptic integrals. Cross sections through the transfer function at different angles, corresponding to different surface tilts, are shown in Fig. 6. One then determines the phase gradient by multiplying this by a linearly increasing function, corresponding to the object spectrum, and finding the projection in the axial direction of the position of the center of gravity. The resulting NA factors are shown in Fig. 7. When the surface tilt is equal to the angular semiaperture of the objective, the NA factor is $1/\cos \alpha$. This is because then the only ray that can be reflected back into the objective is incident normally to the surface.

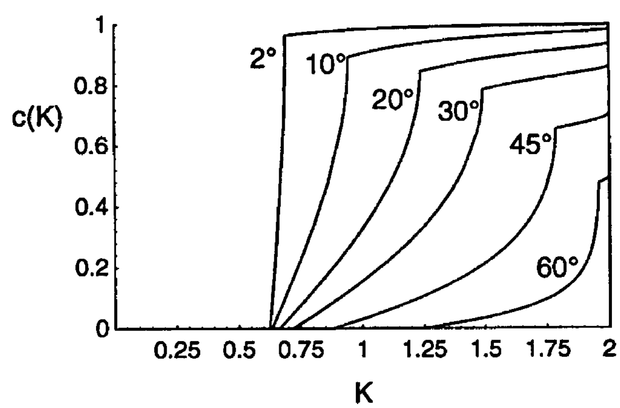

Fig. 6. Cross sections through the coherent transfer function for a system satisfying sine condition NA = 0.95; K is a normalized radial spatial frequency.

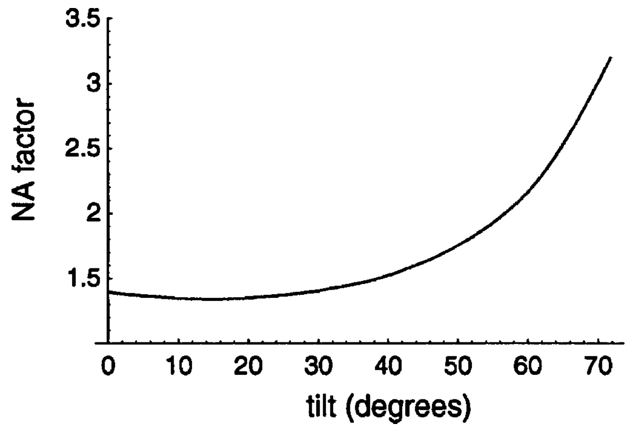

Fig. 7. Effect of surface tilt on the NA factor for a system satisfying sine condition NA = 0.95.

It is interesting to note that the fringe spacing decreases for small tilts and then eventually increases, but it does not change much for tilts smaller than approximately 35° for a NA of 0.95. Through the use of this model, for a level surface it follows immediately that the NA factor for an aplanatic system of numerical aperture unity is 1.5, whereas for a uniform angular illumination it is 2.

5. Discussion

The effects of apodization, central obscuration, and surface tilt on the fringe spacing of interference images have been considered. Rather than being lumped together into an empirical effective numerical aperture, the effects have been investigated separately. The treatment for a level surface is based on a complete electromagnetic-field model, but the effect of tilt is studied by the use of a scalar theory. Therefore, the variation in fringe spacing with tilt for the electromagnetic case may differ for intermediate values of tilt from that presented here.

The theory is based on the evaluation of the phase gradient of the signal beam with axial position. It is applicable to systems in which the interference term is isolated by phase stepping of the reference beam. This can be achieved in the Linnik configuration when the reference mirror is moved together with the matched objective as a unit. If the reference mirror is moved alone,[2] in contrast, then the behavior is much more complicated because the visibility of the fringes varies as the reference beam phase is stepped. This complication is unavoidable in high-NA Mirau configurations, in which the reference mirror is fixed and the specimen is moved. Considerable calibrations difficulties ensue and the use of phase-stepping methods is questionable. Similarly, the behavior of the complete image, including interference and other terms, is also more complicated. In particular, at high aperture only a few fringes are observed within the central lobe of the visibility curve, so that accurate measurements to a fraction of a fringe are difficult without correction for the visibility variation.

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