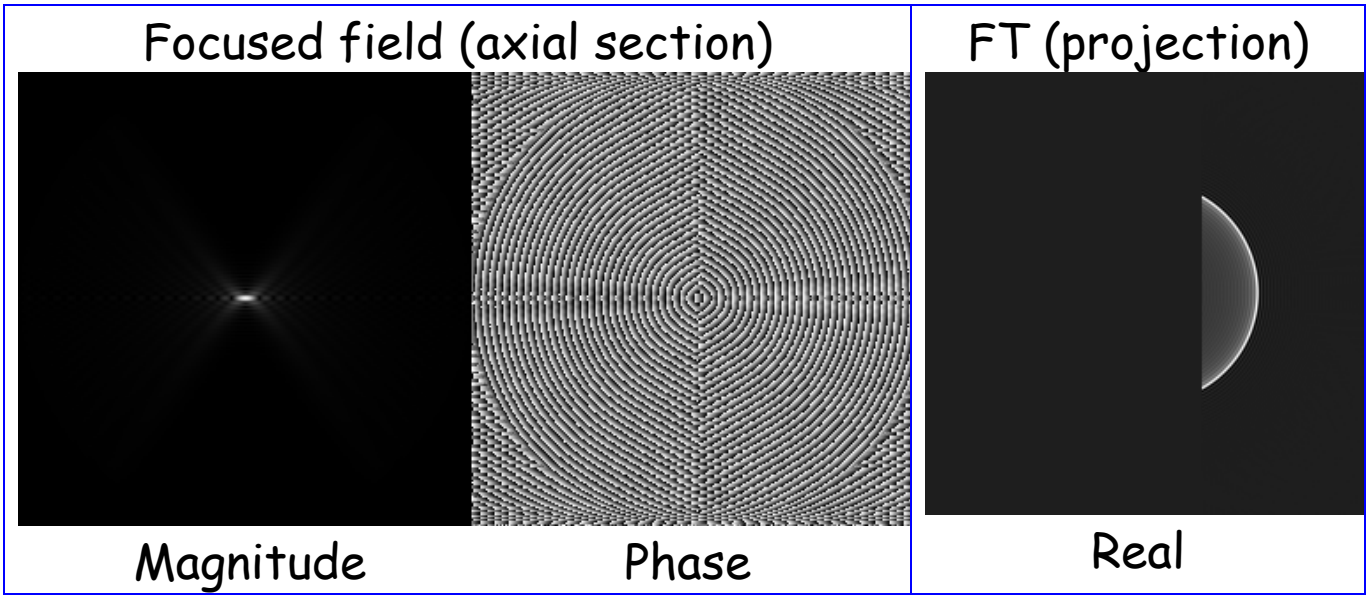
Invariant Large Scale Structure of Axial Diffraction Patterns

Challenge:

- Simple patterns observed in axial plots of intensity.
- No analytic description of patterns known.
- Find (simple) description of patterns.

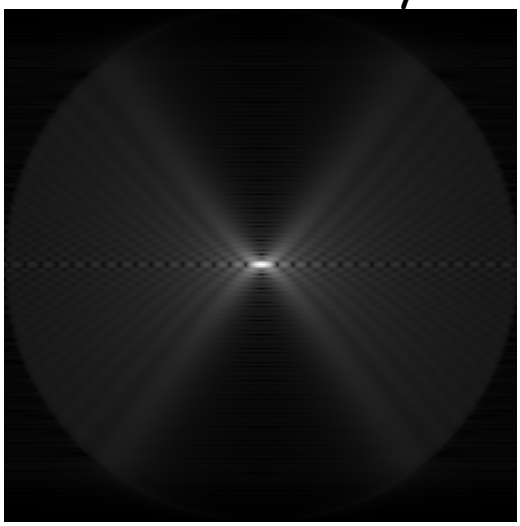
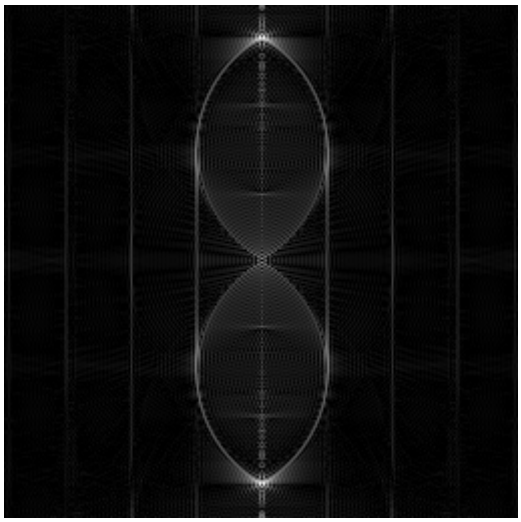
Background

- Debye scalar wave approximation for focused wave.
- Helmholz equation and Fourier transform (FT).
- Axial plane is a SLICE $\Rightarrow$ FT is projection of sphere cap.



Observation

- FT of enhanced PSF has sparse curved line structure

Enhanced intensity PSF	FT
	
Magnitude	Real symmetric

- But the analytic FT of (log) enhanced PSF is intractable.
- Intensity has inverse square drop-off with distance.
- Why not use r^2 enhancement instead? Then FT relation is Laplacian:

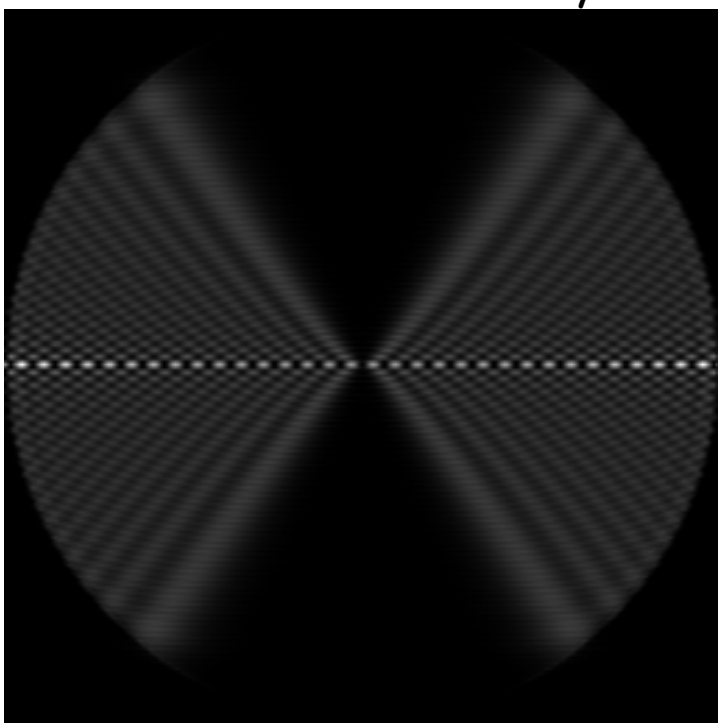
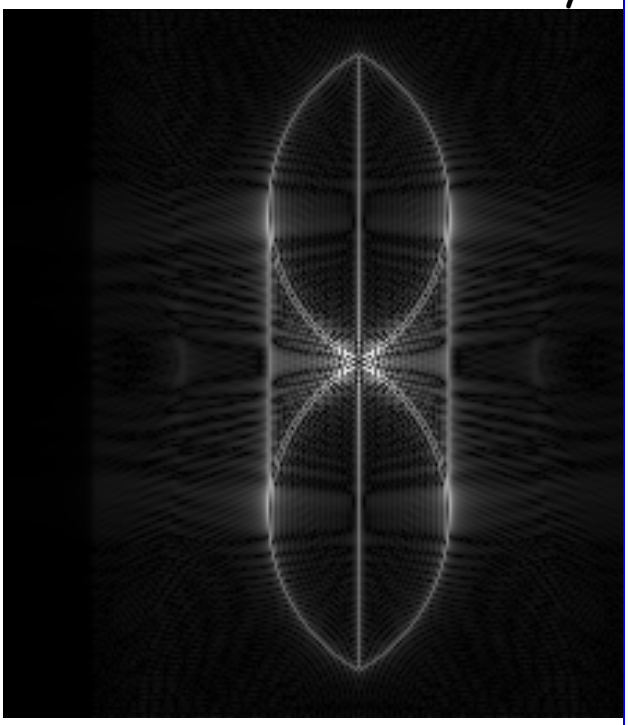
$$\text{PSF} \Leftrightarrow \text{OTF}$$

$$p(x,z) \Leftrightarrow P(u,w)$$

$$-(2\pi r)^2 p(x,z) \Leftrightarrow \nabla^2 P(u,w)$$

- Laplacian picks out edges and discontinuities of OTF.

Enhanced Intensity PSF and Corresponding OTF

r^2 enhanced intensity	FT of enhanced intensity
	
real, positive	only magnitude shown

- Enhancement has interesting feature along axis (horizontal).
- Corresponding ∇^2OTF has vertical lines visible.
- Not really interested in the component along the axis (which is generated by the aperture edge).
- Large scale structure now apparent. Look like overlapping parabolas.
- FT of this structure consists of arcs of circles with dipole-like line cross section.

Initial model of structure

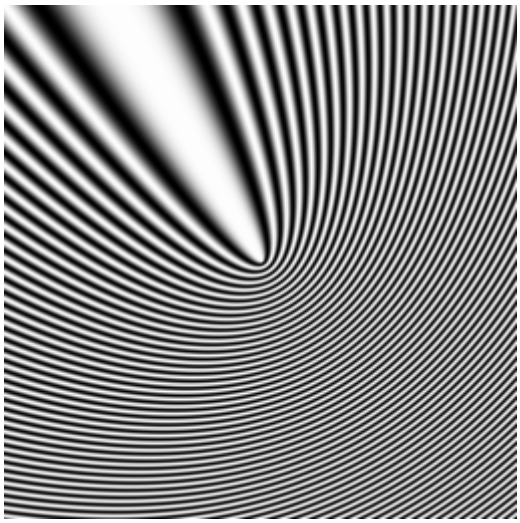
- Consider curvilinear cosine in polar coordinates:

$$p(x,z) = a_1(r)a_2(\theta)\cos[2\pi r f(\theta)]$$
$$x = r\sin(\theta), \quad z = r\sin(\theta)$$

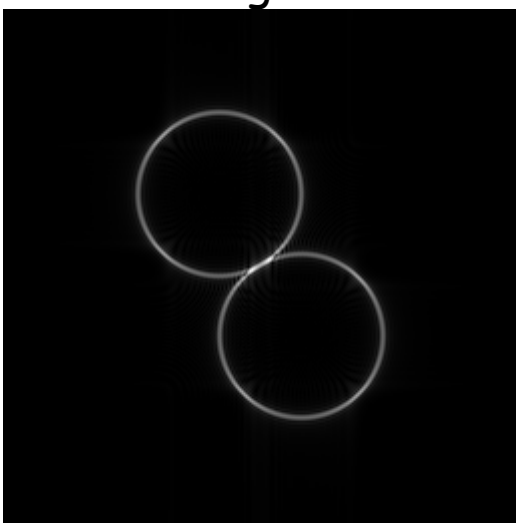
- Particular solutions known (see Amidor for more). Eccentric parabolic cosine:

$$f(\theta) = 1 + \cos(\theta + \beta)$$
$$a_1 = a_2 = 1$$

Eccentric Parabolic Cosine



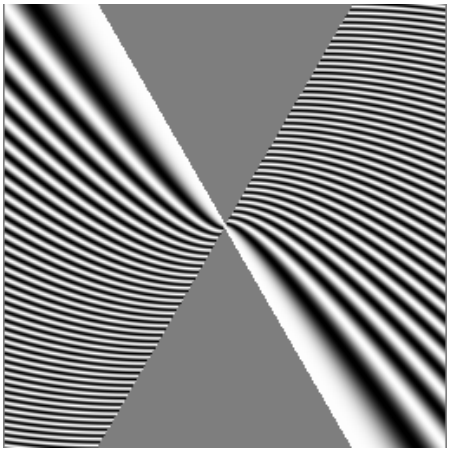
FT Magnitude



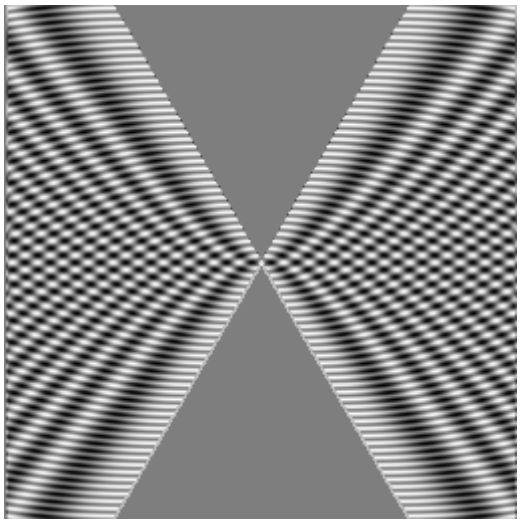
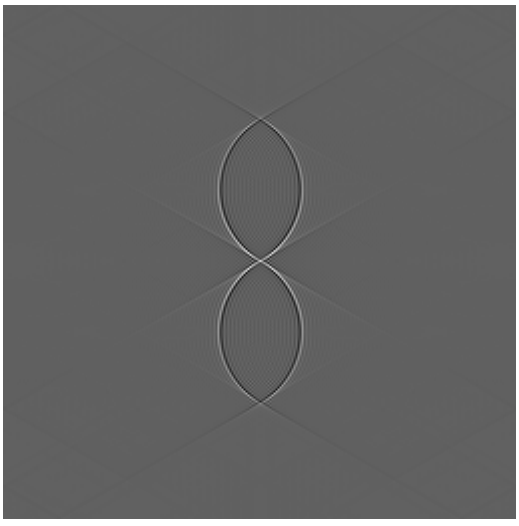
- Analytic solution is two osculating circles with half-order derivative profiles.

Synthesis of simple function with a similar Fourier spectrum

- Modify symmetry of parabolic cosine with shadow edge:

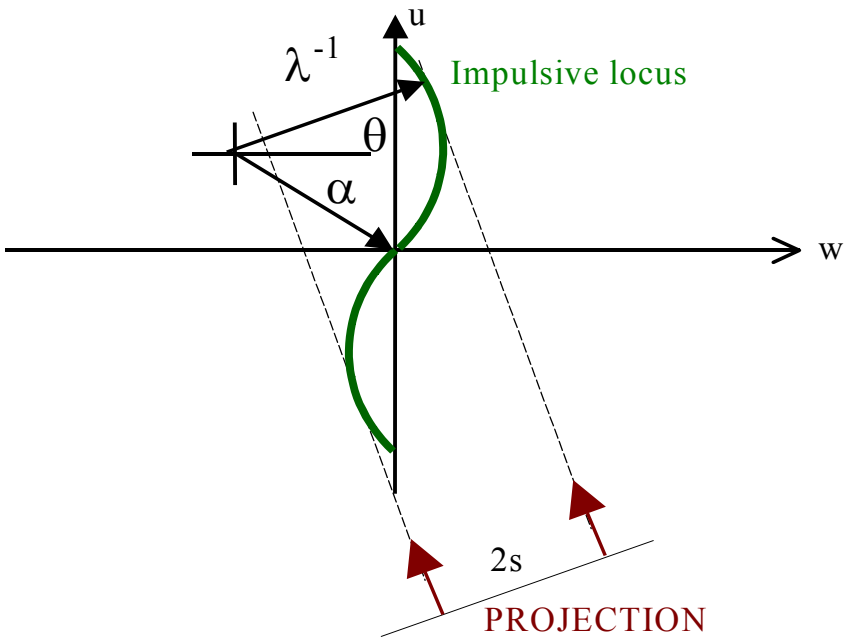
Modified Cosine	FT	
		
real, 180° symmetry	real	imaginary

- Add reflected function to fill in the other Fourier quadrants

Function + Reflection	FT
	
real, symmetric	real

- Synthesis complete, now compare with computed intensities.

Projection-Slice



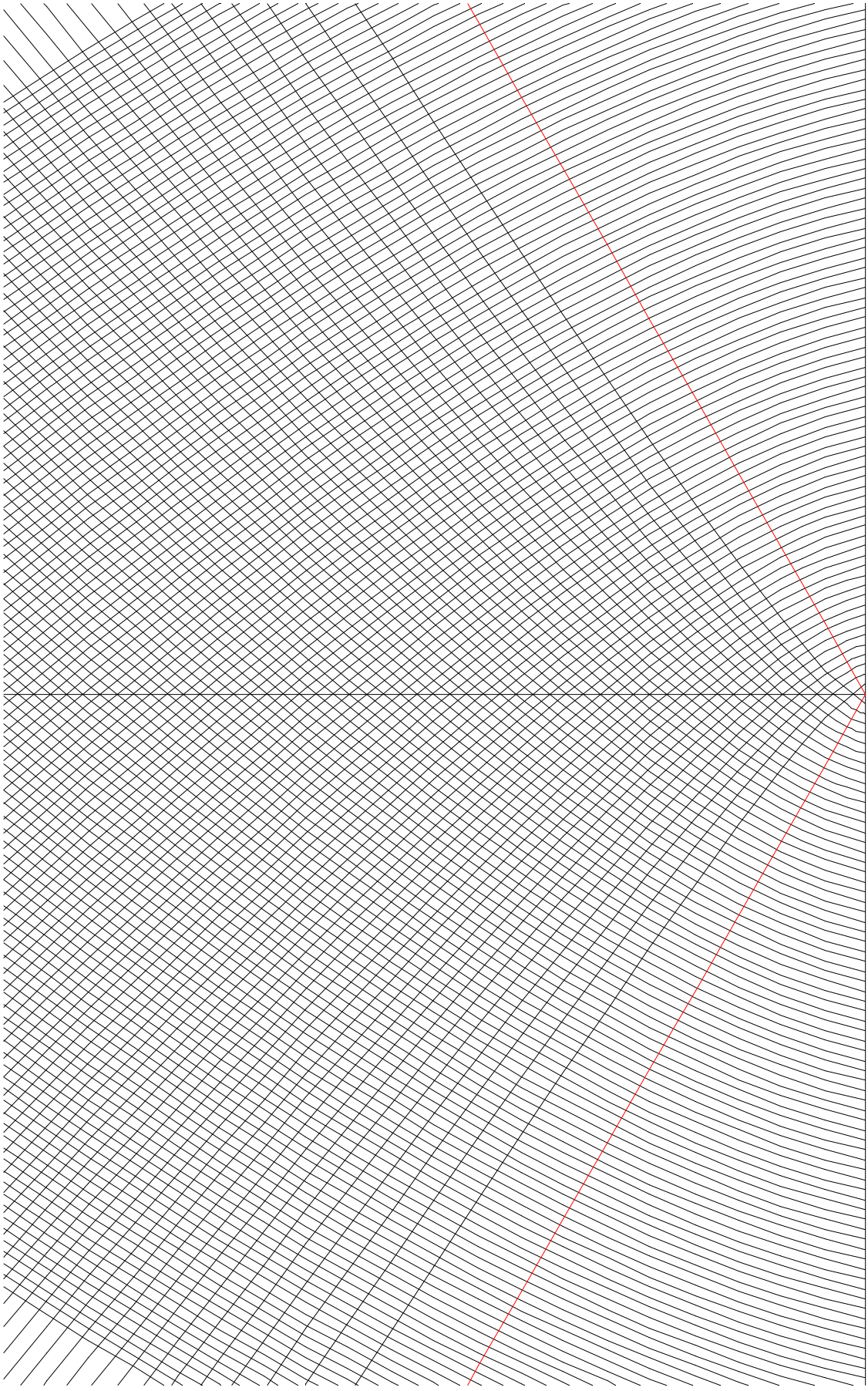
- Projection at θ (Radon) ⇔ Slice of intensity at θ

$$s^{\pm} = \lambda^{-1}\left[1 - \cos(\alpha \pm \theta)\right]$$

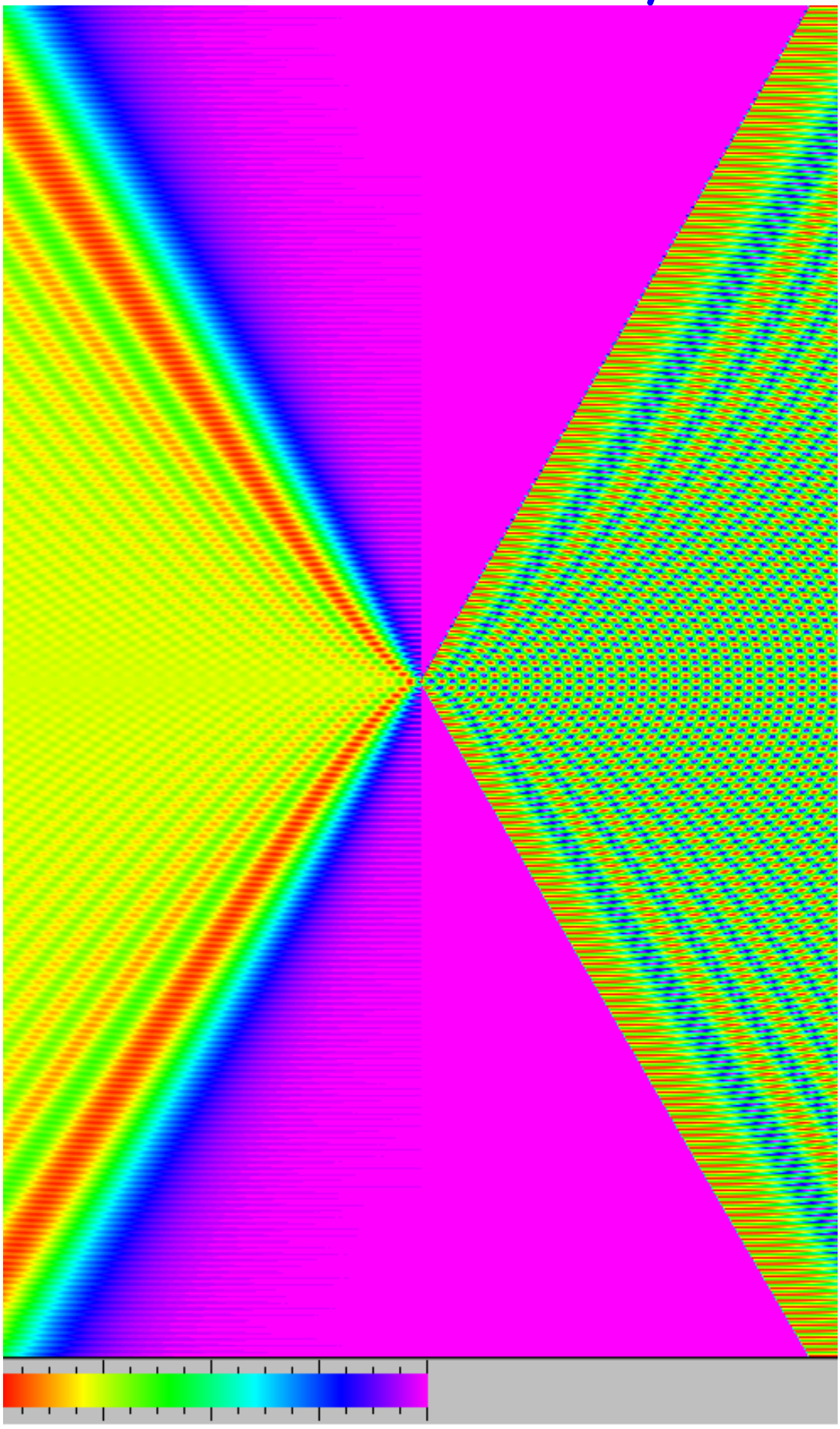
- Adding mirror image loci gives enhanced intensity composed of overlapping cosine patterns:

$$r^2 I(x,y) \approx const.\left[\cos\left(2\pi r s^{+}\right) + \cos\left(2\pi r s^{-}\right)\right]$$

Intensity loci for 0.5 NA System



Diffraction calculation vs Synthesis



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