Extreme Compression of Fingerprint Images: Squeezing Fingerprints until the Spirals Pop Out

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Abstract. We propose a new mathematical model for human fingerprint images. The model can be summarized by the phrase "fingerprints are holograms". The model unifies the analysis, compression, classification, matching, and re-synthesis of fingerprints, in a self-consistent formalism. The parsimony of this model is demonstrated by the reconstruction of fingerprint images with extreme compression ratios (typically >200x). At the heart of the method is a recently proposed method for demodulating two-dimensional fringe patterns, such as holograms. Demodulation uses a spiral-phase quadrature transform combined with a two-dimensional orientation estimator that also uses spiral-phase Fourier operators. Finally, the fingerprint decomposition itself achieves compactness by splitting the phase modulation into two unique parts, one of which is a pure spiral-phase function. Spiral-phase inexorably emerges as a central theme of the work.

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INTRODUCTION

The similarities between optical interferograms and fingerprint patterns have been known for some time. In particular the ridge endings and bifurcations (known as minutiae in the science of fingerprints) resemble the dislocations in wave trains [1] of Nye and Berry. Subsequently these dislocations have been found to be ubiquitous in scalar and vector fields, giving rise to a new branch of research into phase vortex spirals. Much of the intriguing mathematics behind the formation of optical vortices has been developed over the last quarter century. Unfortunately the connection between minutiae and phase spirals does not yet seem to have been generally realized by fingerprint researchers [2].

In parallel to the optical interferometry work there have been a few key publications on oriented patterns, primarily from point of view of topology [3] or image processing [4, 5]. Essentially these works have shown that oriented patterns can be uniquely decomposed into a small number of topologically distinct discontinuities (such as the minutiae) and a well defined smooth flow field. In the area of 2-D phase unwrapping analogous results have been obtained using the concept of residues or phase singularities [6, 7].

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A fusion of the preceding ideas has allowed us to develop a compact formalism capable of representing general oriented patterns.

MODULATION MODEL

The model represents the intensity of a fingerprint image as an amplitude and frequency (phase) modulated function. The equation defining the model is also the general equation for the interference of two coherent beams; a hologram in other words:

$$f(x,y) = a(x,y) + b(x,y).\cos[\psi(x,y)] \tag{1}$$

In practice the above equation may have additional harmonic components as well as many and various types of noise. It turns out that these are important for certain applications, but here fade away when processed by suitably robust mathematical operators (as we shall see). The above formulation is unambiguous if the offset $a(x,y)$, the amplitude $b(x,y)$, and the phase $\psi(x,y)$ are suitably smooth functions. The AM-FM approach has been tried before, but with a dominant frequency that is locally distorted by curvilinear co-ordinates [4], or more recently by a locally defined surface wave [8]. We take a less restrictive approach based on the crucial observation that the main minutiae – ridge ending and bifurcations – can be simply represented by spiral phases of either positive or negative polarity. It turns out that this observation can be formalized by the Helmholtz Decomposition Theorem [HDT]. Usually this is restricted to vector fields, but it can also be applied to the phase representation [7] where the phase is interpreted as a potential function.

MINUTIAE ARE SPIRAL PHASES

The HDT allows us to uniquely decompose the phase $\psi(x,y)$ into two parts. The first part is what we call the smooth phase. The second part is what we call the spiral phase component:

$$\psi(x,y) = \psi_I(x,y) + \psi_R(x,y) \tag{2}$$

The smooth part is also known as the irrotational (or curl-free) component $\psi_I(x,y)$, whilst the spiral phase is also known as the rotational (or divergence free) component $\psi_R(x,y)$. All this is now well known in two-dimensional phase unwrapping theory, thanks to Ghiglia and Pritt's classic text [7]. It is quite easy to artificially create fringe patterns that look very like fingerprint patterns using Eqs. (1) and (2). Multiple minutiae are generated from a spiral phase as follows:

$$\psi_R(x,y) = \sum_{n=1}^{N} p_n \arctan\left(\frac{y-y_n}{x-x_n}\right). \qquad (3)$$

The polarity of each minutia is given by $p_n = \pm 1$, and location by (x_n, y_n). Note that the arctangent needs to be the modulo 2π functional form (for example ATAN2 in the C programming language).

SPIRAL PHASE DEMODULATOR

Equations (1), (2) and (3) allow the simple synthesis of intricate fingerprint patterns. Fig.1 shows a simple synthesized fingerprint loop with 4 minutiae. For the compression task we need to reliably estimate the offset, amplitude and phase in Eq. (1). Until recently this task has been exceedingly difficult owing to the absence of a truly isotropic demodulator in 2-D. Recently a robust method of 2-D demodulation has been proposed [9]. The method is known as spiral phase or vortex demodulation and effectively generalizes the Hilbert transform from 1-D to 2-D.

FIGURE 1. Simple synthesized fringe pattern. Note the dominant loop structure and the ridge ending and bifurcations.

We have previously established [10] that the cosine term Eq. (1) is converted to a sine by the application of a spiral (Fourier) phase operator:

$$\mathcal{S}\{b.\cos\psi\} \cong -ie^{i\beta}.b.\sin\psi. \qquad (4)$$

Orientation and Direction Estimation

A fringe direction (modulo 2π) appears as a phase in Eq. (4). Conveniently the fringe orientation (modulo π) can itself be isotropically estimated by a 2-D energy operator that utilises both first and second order spiral phases [11].

$$\mathbf{E}\{g\} \equiv (\mathbf{\$}\{g\})^2 - g.\mathbf{\$}^2\{g\}. \tag{5}$$

$$\mathbf{E}\{b.\cos\psi\} \cong b^2 \omega^2 e^{2i\beta}. \tag{6}$$

Note that orientation fields have been proposed as a matching feature for fingerprints. It is necessary to unwrap the direction phase from the orientation phase given in Eq. (6). Orientation phase can be unwrapped by halving the double angle estimate and adding π as required (to satisfy local smoothness) as follows:

$$e^{i\beta_u} = \pm\sqrt{e^{2i\beta}} = e^{i(\beta + \pi[1\mp 1]/2)}. \tag{7}$$

More sophisticated unwrapping techniques, using topological properties of the ridge flow fields [12, 13], are also possible. Now we are in a position to generate the complex demodulated image using:

$$b.e^{i\psi} \cong b.\cos\psi - ie^{-i\beta_u}.\mathbf{\$}\{b.\cos\psi\}. \tag{8}$$

Helmholtz Decomposition

At this point in the process we have an estimate of the amplitude and phase modulations from the modulus and argument of the complex function in Eq. (8). Now we can apply conventional 2-D phase unwrapping theory. We prefer to use the classical residue detector of Bone [14] to find the location and polarity of all the spiral phases in the demodulated image. Alternatively the Helmholtz decomposition theorem can be used directly. We then subtract all the spiral phases from the total phase which leaves the smooth phase:

$$\psi_I = \psi - \sum_{n=1}^{N} p_n \operatorname{atan}\left(\frac{y - y_n}{x - x_n}\right) \tag{9}$$

With the spiral phases removed it is trivial to unwrap the smooth phase ψ_I. In practice we find that spiral phase pairs (dipoles) appear because of quantisation and other noise so that the unwrapping has to be a little more sophisticated.

FULLY AUTOMATIC COMPRESSION ALGORITHM

At this point the analysis is complete and it is possible to re-synthesize the original fingerprint $f(x,y)$ from the four component images as follows:

$$f = a + b.\cos[\psi_I + \psi_R]. \qquad (10)$$

The first three images a, b, and ψ_I are all relative smooth functions and each can be compressed considerably with little resultant loss in fingerprint detail. We have not optimized each of the four component compression algorithms, and note that there is plenty of scope for improvement using wavelet or other compression techniques. In this implementation we compress images a, b, and ψ_I using an FFT transform followed by arithmetic coding of the quantized Fourier coefficients. The last image ψ_R is sparse and is encoded either as N vectors, each containing 18 position bits (for a 512^2 image) and one polarity bit for each of the N spirals. Alternatively ψ_R can be compressed directly by run length encoding of the full 2 bit polarity image.

Fig. 2 shows an example of a digitized fingerprint image, 512^2 pixels, with one byte per pixel (262144 bytes total). The fingerprint is taken from the NIST database [15]. Fig. 3 shows the decompressed offset image (52 bytes total). Fig. 4 shows the decompressed amplitude images (25 bytes). Fig. 5 shows the reconstructed smooth phase image (534 bytes).

Fig. 6 shows the decompressed spiral phase image (484 bytes). Fig. 7 shows the overall decompressed fingerprint image (1095 bytes).

FIGURE 2. Digital fingerprint image from NIST database (262,144 bytes).

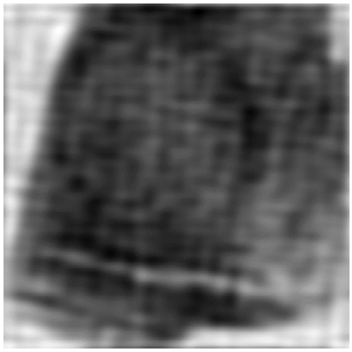

FIGURE 3. Offset image *a(x,y)*, 52 bytes.

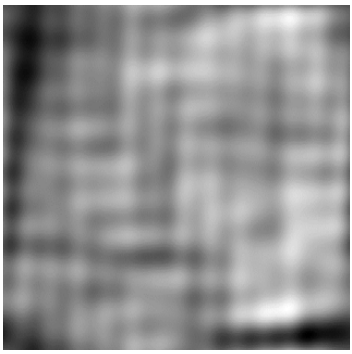

FIGURE 4. Amplitude modulation image *b(x,y)* 25 bytes.

FIGURE 5. Unwrapped smooth phase image $\psi_l(x,y)$, 534 bytes

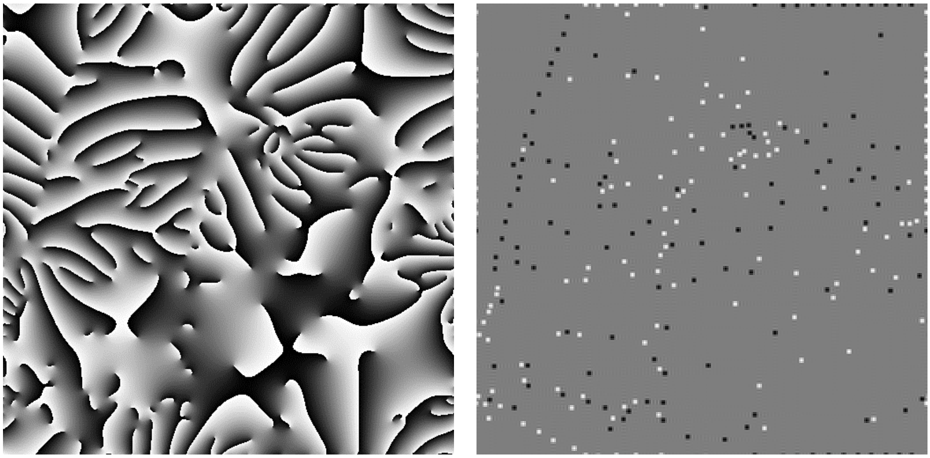

FIGURE 6. Spiral phase image $\psi_R(x,y)$ (left) and its spiral polarity map (right), 484 bytes.

FIGURE 7. Decompressed (239x) fingerprint (1095 bytes) on the right alongside the original (left).

DISCUSSION

We have shown that a modulation based model of fingerprint images allows the image to be split into four elemental sub-images. Each highly redundant sub-image can be compressed drastically using conventional methods. The compressed phase images contain the essential matching and classification features. The phase gradient encodes the orientation and ridge frequency whilst the spiral phases encode the minutiae. Compression ratios greater than 100x are readily attained; far exceeding the typical 15x of the FBI WSQ standard [16]. Further research is needed to quantify the reconstructed image fidelity and visual quality as well as to optimize compression of each component. Nevertheless our approach has the potential to unite the currently disparate strands of fingerprint research in one consistent mathematical model. Note that the entire analysis-compression-synthesis process requires no manual intervention and can be fully automated.

Spiral phase is a recurring theme in this work. Firstly the demodulator uses a spiral phase transform. Secondly the orientation estimator uses two spiral phase transforms. Thirdly, and finally, the phase modulation is made tractable by uniquely splitting it into a smooth phase part and a spiral phase part. At a deeper level the spiral phase represents certain symmetry properties of both the operators and the model itself.

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