UNSW Research Workshop
Function Spaces and Applications,
December 2-6, 2008.

Venue

Room 3085,
Red Centre Building,
School of Mathematics and Statistics,
University of New South Wales,
Kensington, 2052,
Sydney, NSW,
Australia.

Schedule

Date **Time**	**Tuesday** **02/12/08**	**Wednesday** **03/12/08**	**Thursday** **04/12/08**	**Friday** **05/12/08**	**Saturday** **06/12/08**
9:00-10:00	Dooley	Walter	Besov	Kashin	Leinert
10:10-11:10	Larkin	Taubman	Burenkov	Faierman	Ajiev
11:15-12:15	Doust	Duong	Stepanov	Fletcher	lunch
12:20-13:20	lunch	lunch	Wahlberg	lunch	excursion
13:20-14:20	Meaney	McCoy	lunch	Taggart	excursion
14:25-15:25	Ignjatovic	Le Gia		Ji Li	excursion

Which function spaces do fingerprints inhabit? And why should we care?

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Abstract Over the last 40 years many attempts have been made to find an effective mathematical representation for human fingerprint images. Most attempts have failed. The main reason for failure is clear in retrospect.

The most promising representations have been based on AM-FM models (Amplitude Modulation-Frequency Modulation). The concept of instantaneous frequency in two-dimensions is central to such models. Unfortunately the instantaneous frequency contains numerous singularities which cannot be easily contained within such a modulation model.

As an alternative we propose a phase-modulation model to tame the singularities. Once phase is chosen as the basis a very simple functional form can be defined for a fingerprint image:

$$f(\mathbf{r}) = a(\mathbf{r}) + b(\mathbf{r})\cos[\Psi(\mathbf{r})0$$

It turns out that most of the information in a fingerprint image is contained in the phase term. An essential property of the phase is that it is not differentiable at all the important fingerprint matching points, also known as minutiae (or ridge endings and bifurcations). Although some researchers have recently recognised that fingerprint synthesis was possible using the above phase modulation model, it was not until 2006 that a corresponding analysis technique was announced to complete the process.

The heart of the analysis process consists of three main operations:

- isotropic demodulation using the Riesz transform
- isotropic direction estimation using a nonlocal energy operator combined with phase unwrapping
- unique phase separation using the Helmholtz-Hodge decomposition theorem:

$$\psi(\mathbf{r}) \equiv \psi_1(\mathbf{r}) + \psi_2(\mathbf{r})$$

The last step can be considered as a potential function representation of the underlying vector fields. It turns out that one phase component represents the curl-free field, whilst the other represents the divergence-free field. The former encodes the large scale structure of a fingerprint (whorls, loops, arches etc), the latter encodes minutiae - the quintessential matching features.

The phase modulation model splits an image into four elemental sub-images $a(\mathbf{r}), b(\mathbf{r}), \psi_1(\mathbf{r}), \psi_2(\mathbf{r})$. Each of the sub-images is sparse and can be represented by a small amount of (digital) information. Conventional (wavelet) methods of fingerprint image compression attain compression factors of about 15. The new method has been shown to provide at least an order of magnitude improvement (a compression factor of 200 or greater). An additional benefit is that fingerprint matching can occur in the compressed domain: the sparse data directly represents the significant visual features!

In this talk we will present the techniques required to make the fingerprint representation work, as well as showing some examples.