Optik 113, No. 12 (2003) 548–550

http://www.urbanfischer.de/journals/optik

Optik Optics
International Journal for Light and Electron Optics

Information capacity and resolution in three-dimensional imaging

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Abstract: In an imaging system, it is known that the information capacity, rather than the resolution, is invariant. Thus super-resolution can be achieved by encoding/decoding additional information on to independent parameters of the imaging system. The concept of information capacity is extended to the full three-dimensional case.

Key words: Information capacity – optical resolution – polarization – optical transfer functions

1. Introduction

Recently, a remarkable paper [1] discussed the possibility of increasing the information capacity of a communication channel by utilising the polarization of electromagnetic waves in three-dimensional space. Since there is an analogy between the behaviour of a communications channel and an imaging system, i.e. an imaging system can be considered as a channel transmitting spatial information; this suggests that similar results are applicable to the information capacity of an imaging system. This paper extends previous treatments of information capacity and resolution of optical systems to the full three-dimensional regime. This is particularly relevant when it is appreciated that three-dimensional imaging is of great topical interest, in areas such as confocal microscopy, two-photon fluorescence microscopy, optical coherence tomography, and image deconvolution.

2. Information capacity in 2-D images

Information capacity, in bits, was defined by Hartley [2] as

$$N = \log_2 m\,, \tag{1}$$

where m is the total number of possible message states.

The concept of the channel capacity of a temporal communications system was introduced by Shannon [3]. For a signal of duration T and a channel of bandwidth B_T, the number of points M needed to specify the system is

$$M = 2TB_T + 1\,, \tag{2}$$

where the unity term accounts for the fact that a dc signal can be measured with a channel of zero bandwidth. In communications, this is usually neglected, as the other term is much greater than unity.

If the detected signal has an average power s and an additive noise power n, assumed band-limited and uncorrelated, the number of levels that can be distinguished is given by

$$[(s+n)/n]^{1/2}\,, \tag{3}$$

so that the total number of possible message states is

$$m = [(s+n)/n]^{M/2}\,. \tag{4}$$

Thus the information capacity is [3]

$$N = (2TB_T + 1)\,\log_2(1 + s/n)/2\,. \tag{5}$$

Fellgett and Linfoot [4] applied this approach to the information capacity of a two-dimensional imaging system, neglecting the dc terms. This gives

$$\begin{aligned} N &= 2B_xL_x2B_yL_y\,\log_2[1+s/n]^{1/2} \\ &= 2B_xL_xB_yL_y\,\log_2[1+s/n]\,, \end{aligned} \tag{6}$$

where the value at each point of the object is real for an incoherent system and complex for a coherent system [5]. Lukosz proposed [6] that all superresolution techniques can be explained by the theorem of invariance of the number of degrees of freedom N_F of an imaging system, where

$$N_F = 2(2L_xB_x + 1)\,(2L_yB_y + 1)\,(2TB_T + 1)\,, \tag{7}$$

in which B_x, B_y are the spatial bandwidths, L_x, L_y are the dimensions of the field of view in the x, y direc-

Received 23 September 2002; accepted 20 December 2002.

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tions respectively, and the factor 2 accounts for two polarization states. It is noted that Lukosz included the temporal effects of the observation time T, but neglected the effects of noise. Lukosz argued that superresolution can be achieved by trading off another of the constituent parameters of eq. 7, which requires that some *a priori* information concerning the object is known. Cox and Sheppard [5] combined the results of Fellgett and Linfoot, and of Lukosz, to include the dc terms, time dependence, noise, and two independent states of polarization, so that the information capacity,

$$N = (2L_xB_x + 1)\,(2L_yB_y + 1) \times (2TB_T + 1)\log_2(1 + s/n)\,, \tag{8}$$

is invariant. They went on to analyse the performance of the particular case of superresolution by analytic continuation, and also to discuss superresolution by confocal imaging.

3. Information capacity in 3-D images

Cox and Sheppard also incorporated axial imaging, with L_z the depth of field and B_z the axial spatial bandwidth, but the existence of a defined optic axis was implied by the incorporation of two states of polarization. Considering the results of Andrews *et al.*, therefore, eq. 8 could be revised to include a total of six polarization states, for the electric and magnetic field components in the three directions, to give

$$N = 3(2L_xB_x + 1)\,(2L_yB_y + 1)\,(2L_zB_z + 1) \times (2TB_T + 1)\log_2(1 + s/n)\,. \tag{9}$$

However, this expression assumes that the behaviour is separable into three Cartesian coordinates. An alternative is to consider three-dimensional spatial frequency space, as is traditionally done in calculation of the modes of a black body cavity, or the density of states in condensed matter physics, so that

$$N = 3(8VB_v + 1)\log_2(1 + s/n)\,, \tag{10}$$

where V is the volume of the object and B_v the volume of the three-dimensional transfer function in reciprocal space.

In one dimension, the spatial frequency cut-off of a coherent far-field optical imaging system is equal to $\lambda^{-1}\sin\alpha$ where λ is the wavelength of the radiation, and $2\lambda^{-1}\sin\alpha$ for an incoherent system. Similarly, following previous work, the total volume of the three-dimensional transfer function is calculated as a function of $\sin\alpha$ rather than a weighted volume. For a coherent system, the appropriate coherent transfer function (CTF) was first proposed by Wolf [7]. The CTF is non-zero only on the cap of a sphere of radius λ^{-1}, that subtends an angle α at its centre, analogous to part of the Ewald sphere of X-ray crystallography. Thus for a quasi-monochromatic system of spectral spread $\Delta\lambda$, B_v is given by

$$B_v = \frac{2\pi}{\lambda^4}\,(1 - \cos\alpha)\,\Delta\lambda\,. \tag{11}$$

Because the CTF is effectively given by a surface, three-dimensional imaging of a general object is poor. This also explains how the information of a hologram can be stored on a two-dimensional film [8]. Although the factor of six assumed in eq. 10 allows for the six possible electric and magnetic dipoles generated at the focus, for small apertures this perhaps should be replaced by four, representing two possible transverse states, together with axial electric and magnetic dipoles [9, 10], as transverse electric and magnetic dipoles appear together [11].

The corresponding volumes of the CTF for confocal transmission and reflection systems [12–14] are

$$B_v = \frac{2\pi}{\lambda^3}\sin\alpha(\alpha - \sin\alpha\cos\alpha) \tag{12}$$

and

$$B_v = \frac{16\pi}{3\lambda^3}\left(1 - \cos\alpha - \frac{1}{2}\sin^2\alpha\cos\alpha\right), \tag{13}$$

respectively, and the dependence on $\sin\alpha$ shown in fig. 1. For small α, the two volumes are equal. For a confocal system with complete 4π steradian illumination and collection, $B_v = 32\pi/(3\lambda^3)$.

For a conventional fluorescence system [12–14], the OTF is identical to the CTF for a confocal transmission system, so that eq. 12 holds for B_v. For a confocal fluorescence system, neglecting the Stokes' shift in wavelength, the volume is

$$B_v = \frac{16\pi}{3\lambda^3}\left[\frac{2}{3}(1 - \cos\alpha) - \frac{7}{3}\sin^2\alpha\cos\alpha + \alpha\sin\alpha + \sin^2\alpha\right], \tag{14}$$

These volumes are illustrated in fig. 2. For fluorescence systems of a complete solid angle of 4π steradians,

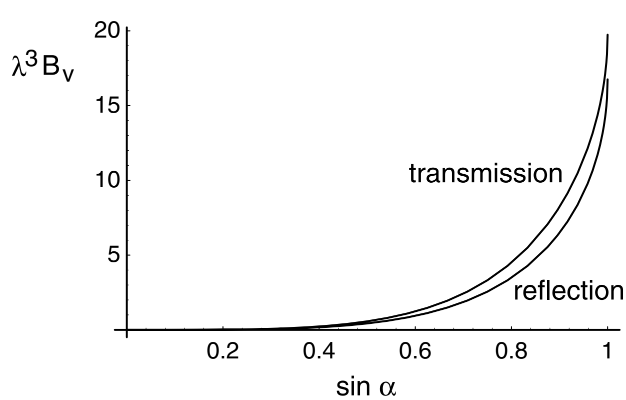

Fig. 1. The volume of the three-dimensional coherent transfer function (CTF) of confocal transmission and reflection systems, as a function of sin α.

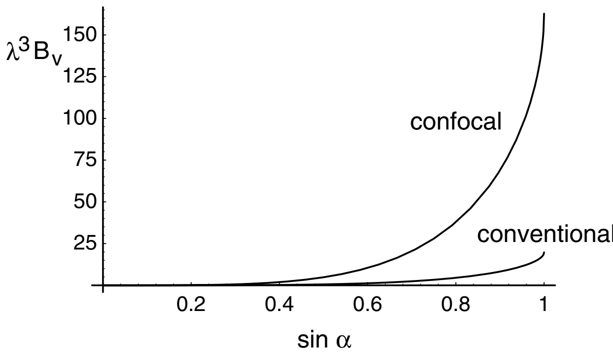

Fig. 2. The volume of the three-dimensional optical transfer function (OTF) of conventional and confocal fluorescence systems, as a function of sin *α*.

$B_v = 32\pi/(3\lambda^3)$ and $B_v = 256\pi/(3\lambda^3)$, respectively, for conventional and confocal systems. However, in fluorescence the polarization factor is three, rather than six, as fluorescent molecules behave as electric dipoles to a first approximation [15].

4. Discussion

The concept of information capacity for a 3-D image with independent magnetic and electric field components may prove useful in the analysis of superresolution schemes, and for comparing resolution capabilities of different optical systems. The information capacity is essentially proportional to the product of the 3-D optical transfer function volume, the image volume, and the number of independent components. Future refinements of the model should include the spectral properties of the noise thus substantiating the influence of the transfer function magnitude upon the capacity measure. Although the information capacity levels are essentially three times that previously believed, practical methods of generating or detecting all 6 polarization states independently in an optical field are not presently available.

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